

Convexification of Hybrid AC-DC Optimal Power Flow with Line-commutated Converters

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Abstract—Line-commutated converter (LCC)-based high-voltage DC (HVDC) systems have been integrated with bulk AC power grids for interregional transmission of renewable power. The nonlinear LCC model brings additional nonconvexity to optimal power flow (OPF) of hybrid AC-DC power grids. A convexification method for the LCC station model could address such nonconvexity but has rarely been discussed. We devise an equivalent reformulation for classical LCC station models that facilitates second-order cone convex relaxation for the OPF of LCC-based AC-DC power grids. We also propose sufficient conditions for exactness of convex relaxation with its proof. Equivalence of the proposed LCC station models and properties, exactness, and effectiveness of convex relaxation are verified using four numerical simulations. Simulation results demonstrate a globally optimal solution of the original OPF can be efficiently obtained from relaxed model.

Index Terms—AC-DC power system, convex relaxation, line-commutated converter, optimal power flow, second-order cone programming.

NOMENCLATURE

A. Abbreviations

ADOL	Alternating-current and direct-current optimal power flow with line-commutated converter.
ARE	Average relative error.
BFM	Branch flow model.
COS	Constructed optimal solution.
HVDC	High-voltage direct current.
LCC-HVDC	Line-commutated-converter-based HVDC.
MRE	Maximum relative error.
MTDC	Multiterminal direct current.
OPF	Optimal power flow.
POS	Primitive optimal solution.
RESs	Renewable energy resources.
RPCDs	Reactive power compensation devices.
SDP	Semidefinite programming.
SOC	Second-order cone.

SOC	Second-order cone programming.
TTDC	Two-terminal direct current.
VSC-HVDC	Voltage-source-converter-based HVDC.

B. Sets

$\mathbb{C}$	Set of complex numbers.
$\mathbb{N}^{\text{ac}}$	Set of AC buses in an AC-DC grid.
$\mathbb{N}^{\text{dc}}$	Set of DC buses in an AC-DC grid.
$\mathbb{N}^{\text{conv}}$	Set of converter buses in an AC-DC grid.
$\mathbb{E}^{\text{ac}}$	Set of AC lines in an AC-DC grid.
$\mathbb{E}^{\text{dc}}$	Set of DC lines in an AC-DC grid.
$\mathbb{R}$	Set of real numbers.
$\mathbb{X}$	Set of solutions of the ADOL.
$\mathbb{Y}$	Set of solutions of the angle relaxation ADOL.
$\bar{\mathbb{Y}}$	Set of solutions of the SOC relaxation ADOL.

C. Parameters, Variables and Functions

c_{2i}, c_{1i}, c_{0i}	Generation cost coefficients.
$\bar{I}_{ij}$	AC transmission complex current.
I_{di}	Injection current of the DC bus.
I_{dij}	DC transmission current.
φ_{ci}	Converter power factor angle.
k_{Ti}	Converter transformer ratio.
P_{ci}	Injection active power of the converter.
P_{di}	Injection active power of the DC bus.
P_{dij}	DC transmission power.
P_{Gi}	Active power generation.
P_{ij}	AC transmission active power.
P_{Ldi}	DC power load.
P_{Li}	AC active power load.
Q_{ci}	Injection reactive power of the converter.
Q_{cci}	Reactive power compensation.
Q_{Gi}	Reactive power generation.
Q_{ij}	AC transmission reactive power.
Q_{Li}	AC reactive power load.
θ_{ci}	Converter control angle.
θ_i	Phase angle of the AC voltage.
r_{dij}	DC line resistance.
r_{ij}	AC line resistance.
$\text{sign}(\cdot)$	Sign function.
$\bar{U}_i$	AC bus complex voltage.
U_{di}	DC bus voltage.
$\mathbf{x}$	Solution of the ADOL.
x_{ij}	AC line reactance.
x_{Li}	Leakage reactance of the converter transformer.
$\mathbf{y}$	Solution of the relaxed ADOL.

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I. INTRODUCTION

The energy crisis and environmental deterioration have become increasingly serious issues, and developing and utilizing renewable energy resources (RESs) are important trends in global energy development [1]–[3]. High uncertainty of large-scale RESs has brought unprecedented challenges for planning and operation of power grids [4], [5]. High-voltage direct current (HVDC) transmission technology is used for grid connection of large-scale RESs and interconnection of bulk power grids because of its high operational flexibility, low transmission losses, and asynchronous features. Voltage source converter (VSC)-based HVDC systems bridge volatile RESs and power grids with independent control of active and reactive power, resulting in lower transmission losses than AC systems. Line-commutated converter (LCC)-based HVDC technology is well suited for long-distance and interregional power transmission due to its larger transmission capacity and smaller converter losses.

Steady-state models of VSCs and LCCs are described using nonlinear equations. For example, VSC losses are quadratic in currents, and power injection and voltage conversion equations of LCCs involve trigonometric functions. Hence, it is difficult to solve optimal power flow (OPF) of power grids with HVDC integration [6]. In this context, due to the quadratic terms in equalities, the OPF problem is cast as a nonconvex nonlinear programming problem and has been proven to be NP-hard [7], [8].

Linear approximation is an option for the nonlinear OPF. Classical DC power flow [9] significantly reduces computational burden of solving the OPF, but it may be difficult to obtain accurate results for networks with large R/X ratios. In this regard, other types of linear power flow models have been developed, such as the Taylor series expansion-based linear approximation [10]–[12] and its application to different independent variables [13]–[15]. Complex Taylor series expansion was applied in [16] to derive higher-order approximations and further improve accuracy of power flow solution. A polyhedral outer approximation was performed to linearize a relaxed AC-OPF model used to clear distribution power market in [17]. In general, optimal solution of linearized OPF is rarely feasible for the original problem, and a number of linearized models need to be solved sequentially to obtain a fair-quality solution.

As an alternative, convex relaxation is helpful for obtaining a globally optimal solution of the OPF under certain conditions. Second-order cone programming (SOCP) and semidefinite programming (SDP) were recently explored to relax AC OPF problem. In [18], SOCP relaxation was applied to AC branch flow model (BFM) and was proven to provide exact relaxation of the AC OPF problem for radial networks if there are no upper bounds on load demands. Strengthened SOCP relaxations for the AC OPF problem were proposed in [19], [20]. References [21] and [22] formulated the AC OPF problem as an SDP problem. Reference [23] derived the duality of SDP relaxation of the AC OPF problem and provided a sufficient condition for zero duality gap. In [24], exactness of SDP relaxation for weakly cyclic networks was studied, and upper bound on rank of minimum-rank solution

was given. References [25] and [26] summarized different forms, exactness conditions, and strengths of SOCP and SDP relaxation for the AC OPF problem. Generally, SDP relaxation is stronger than SOCP relaxation because SDP relaxation places more stringent requirements on model and can retain more information about original variables and constraints. Other types of relaxation methods can be found in [27]–[29]. The aforementioned convex relaxation methods have been widely used in power system scheduling and operation optimization problems [30]–[32].

Convex relaxation techniques devised in the above references focus on handling nonconvexity relating to load flow equations in OPF. They are not directly applicable to hybrid AC-DC power grid because nonlinear equations of converters are not addressed. In [33], the SOCP relaxation of AC-DC OPF based on BFM was explored by formulating constraints of VSC station as quadratic and affine. Reference [34] suggested placing one AC line in every cycle of mesh network with an HVDC line to facilitate exact SDP relaxation of the OPF problem and argued active power on DC branch is transferred with a loss factor. An SDP relaxation of AC-DC OPF was proposed in [35], which introduced a penalty term into the objective function to guarantee feasibility of the relaxed optimal solution but resulted in inexact relaxation. Reference [36] employed convex hulls to relax nonlinear equations of VSC and LCC station models but did not investigate their exactness.

References [33]–[35] discussed AC-DC OPF with VSCs alone, while cases with LCCs were not mentioned. Because modeling of LCCs involves more complicated nonlinear equations than VSCs, OPF with LCCs is more challenging to solve. Reference [36] tried to develop a unified model for both VSCs and LCCs, but converter control angle of LCCs was not included, and equations for converter power factor angle of LCCs were not described. Work on AC-DC OPFs with accurate and classical modeling for LCCs can be found in [38]–[41]. These OPF models were solved by interior point methods [38], [39] and heuristic algorithms [40], [41], but global optimality of the solution cannot be guaranteed. The above classical LCC models involve nonlinear equations with bilinear and trigonometric functions; thus, they bring more variables and constraints to OPF and make OPF nonconvex. One approach to relaxing such a problem is to replace nonlinear terms with their convex envelopes [27], [28], but this does not lead to exact relaxation in general. In addition, although DC power flow model has good performance in transmission system, it does not involve reactive power and voltage and hence does not meet accuracy requirements of LCC station model, which includes reactive power compensation devices and converter transformers. Therefore, AC power flow model must be used, which makes convex relaxation for LCC station model and OPF more difficult and challenging.

In summary, convex relaxation can be effective in obtaining globally optimal solutions to AC-DC OPF with LCCs (ADOL). However, convexification of LCC, a critical component in a hybrid AC-DC system, remains to be investigated. To bridge this gap, we derive a set of quadratic equations to describe the full LCC model and develop a convex relaxation

method for ADOL with proven sufficient conditions for its exactness. The contributions of this paper are threefold:

1) A quadratic reformulation of LCC station model is devised for the first time, in which all quadratic equations are surfaces of second-order cones, making it possible to perform SOCP relaxation for the ADOL.

2) With the above LCC model, the ADOL is formulated as a quadratic-constrained quadratic programming (QCQP) problem and is recast as an SOCP problem by relaxing quadratic equalities for LCCs and networks. Sufficient conditions of such a relaxation, under which globally optimal solution of the original problem can be recovered from the relaxed ADOL solution, are proposed and proved.

3) Simulations are performed on different AC-DC test systems with two-terminal DC (TTDC) and multiterminal DC (MTDC) to validate the proposed LCC station model and to verify properties, exactness, and computational performance of the relaxed ADOL.

The remainder of this paper is organized as follows. In Section II, the proposed LCC station model is developed based upon classical modeling. In Section III, ADOL is formulated, and proposed convex relaxation of the ADOL is illustrated. Sufficient conditions for exactness are discussed in Section IV. In Section V, simulations are conducted to demonstrate performance of the proposed relaxed model. Finally, conclusions are presented in Section VI.

II. FORMULATION OF THE LCC STATION MODEL

In this section, a classical form of the LCC station model is presented. To facilitate further relaxation, it is equivalently reformulated using linear and quadratic equations. All symbols and acronyms are summarized in the Nomenclature section.

A. Classical Form of LCC Station Modeling

As shown in Fig. 1, the LCC station consists of filters, reactive power compensation devices (RPCDs), transformers, and converters. RPCDs, such as shunt capacitors, static synchronous compensators, and static VAR compensators, provide reactive power injection for converter to support voltage profiles. Filters are used to reduce harmonics and generate reactive power. Converter completes the commutation of AC and DC voltages.

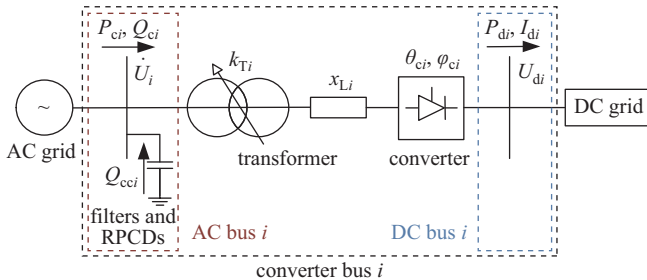


Fig. 1. Typical structure of an LCC station.

The classical LCC station model [37]–[41] is presented as follows. AC bus i and DC bus i connected by converter

station are combined into one converter bus $i \in \mathbb{N}^{\text{conv}}$. Voltage amplitudes on both sides are constrained by

$$U_{di} = k_{Ti} U_i \cos \theta_{ci} - x_{Li} I_{di} \cdot \text{sign}(I_{di}) \quad \forall i \in \mathbb{N}^{\text{conv}} \quad (1)$$

Active power transmitted by converter station is

$$P_{di} = U_{di} I_{di} \quad \forall i \in \mathbb{N}^{\text{conv}} \quad (2)$$

Assuming converter and transformer are lossless, active power balance equation for a converter-connected bus is

$$P_{ci} = P_{di} \quad \forall i \in \mathbb{N}^{\text{conv}} \quad (3)$$

Reactive power consumed by converter is determined by transmitted active power and power factor angle of converter; i.e.,

$$Q_{ci} = \text{sign}(P_{ci}) \cdot P_{ci} \tan \varphi_{ci} \quad \forall i \in \mathbb{N}^{\text{conv}} \quad (4)$$

where power factor angle is calculated by

$$U_{di} = k_{Ti} U_i \cos \varphi_{ci} \quad \forall i \in \mathbb{N}^{\text{conv}} \quad (5)$$

Control angle and converter transformer ratio have the following limits:

$$\underline{\theta}_{ci} \leq \theta_{ci} \leq \bar{\theta}_{ci} \quad \forall i \in \mathbb{N}^{\text{conv}} \quad (6)$$

$$\underline{k}_{Ti} \leq k_{Ti} \leq \bar{k}_{Ti} \quad \forall i \in \mathbb{N}^{\text{conv}} \quad (7)$$

Power losses of the LCC station are omitted in the classical LCC station model, as are losses of the proposed LCC station model. This is because power losses of the LCC station are negligibly small, i.e., typically less than 1% of total power transferred [42], [43].

In addition, RPCDs are considered in the proposed converter model. The LCC is a reactive power load for AC power grid, reactive power consumption of which equals approximately 40%–60% of transmitted active power [44], [45]. Thus, RPCDs are generally installed in the LCC station to support reactive power balance locally. In this paper, reactive power outputs of the RPCDs are among decision variables in OPF.

B. Reformulation

Equations (1), (4), and (5) are nonlinear in variables θ_{ci} , k_{Ti} , and φ_{ci} with bilinear and trigonometric functions, and they are difficult to convexify. Therefore, it is necessary to reformulate the LCC station model.

In fact, (1) can be cast as a linear equality as follows. Observing k_{Ti} appears only in the product with U_i in (1) and (5), let

$$v'_{si} = k_{Ti}^2 U_i^2 \quad \forall i \in \mathbb{N}^{\text{conv}} \quad (8)$$

$$v''_{si} = v'_{si} \cos^2 \theta_{ci} \quad \forall i \in \mathbb{N}^{\text{conv}} \quad (9)$$

From (1), we have $U_{di} + x_{Li} I_{di} \cdot \text{sign}(I_{di}) = k_{Ti} U_i \cos \theta_{ci}$. Taking magnitude squared and substituting (8) and (9) into it, we obtain $U_{di}^2 + 2x_{Li} P_{di} \cdot \text{sign}(P_{di}) + x_{Li}^2 I_{di}^2 = v''_{si}$. Letting $v_{di} = U_{di}^2$ and $\ell_{di} = I_{di}^2$, we have

$$v''_{si} - v_{di} = 2x_{Li} P_{di} \cdot \text{sign}(P_{di}) + x_{Li}^2 \ell_{di} \quad \forall i \in \mathbb{N}^{\text{conv}} \quad (10)$$

Accordingly, limit constraints (6)–(7) are modified as follows:

$$v'_{si} \cos^2 \underline{\theta}_{ci} \leq v''_{si} \leq v'_{si} \cos^2 \bar{\theta}_{ci} \quad \forall i \in \mathbb{N}^{\text{conv}} \quad (11)$$

$$\underline{k}_{Ti}^2 v_i \leq v_{si}' \leq \bar{k}_{Ti}^2 v_i \quad \forall i \in \mathbb{N}^{\text{conv}} \quad (12)$$

where $\underline{\cos^2 \theta_{ci}}$ and $\overline{\cos^2 \theta_{ci}}$ denote the minimum and maximum $\cos^2 \theta_{ci}$ with $\theta_{ci} \in [\underline{\theta}_{ci}, \bar{\theta}_{ci}]$, respectively.

Equations (1) and (5)–(7) are replaced with (10)–(12) to eliminate θ_{ci} and k_{Ti} . In addition, (4) and (5) can be cast into a quadratic equality. Substituting (8) into (5), we have

$$v_{di} = v_{si}' \cos^2 \varphi_{ci} \quad \forall i \in \mathbb{N}^{\text{conv}} \quad (13)$$

By squaring both sides of (4) and (2), we obtain

$$P_{ci}^2 + Q_{ci}^2 = P_{ci}^2 (\tan^2 \varphi_{ci} + 1) = P_{ci}^2 / \cos^2 \varphi_{ci} \quad \forall i \in \mathbb{N}^{\text{conv}} \quad (14)$$

$$P_{di}^2 = v_{di} \ell_{di} \quad \forall i \in \mathbb{N}^{\text{conv}} \quad (15)$$

Substituting (3), (13) and (15) into (14) yields

$$P_{ci}^2 + Q_{ci}^2 = v_{si}' \ell_{di} \quad \forall i \in \mathbb{N}^{\text{conv}} \quad (16)$$

Equality (16) is an alternative to (4) and (13) that eliminates φ_{ci} . For $\varphi_{ci} \in (0, \pi)$, the right side of (4) is nonnegative, indicating that LCC is always inductive to AC grid [45]. Hence, the following constraint should be added:

$$Q_{ci} \geq 0 \quad \forall i \in \mathbb{N}^{\text{conv}} \quad (17)$$

In summary, reformulated model of an LCC station is expressed by linear equalities (3) and (10), linear inequalities (11), (12) and (17), and quadratic equalities (15) and (16) with new variables v_{si}' and v_{si}'' . Values of eliminated variables θ_{ci} , k_{Ti} , and φ_{ci} can be recovered by (8), (9) and (13). In addition, quadratic equalities (15) and (16) can be relaxed to inequalities as second-order cones (SOCs). In Section III, this model will be applied in the ADOL for further convexification.

III. FORMULATION AND CONVEXIFICATION OF THE ADOL

In this section, we first formulate original ADOL using BFM of AC and DC grids and the classical LCC station model described in Section II-A. Then, we perform SOCP relaxation to the ADOL based on reformulated LCC station model.

A. Formulation of the ADOL

The ADOL is described as

$$\text{ADOL:} \quad \min_{\mathbf{x}} \{f(\mathbf{x}) | \mathbf{x} \in \mathbb{X}\} \quad (18)$$

and the details are as follows.

1) Objective Function

Objective function $f(\mathbf{x})$ is typically total generation cost:

$$f(\mathbf{x}) = \sum_{i \in \mathbb{N}^{\text{ac}}} c_{2i} P_{Gi}^2 + c_{1i} P_{Gi} + c_{0i} \quad (19)$$

2) Decision Variables

$\mathbf{x} = (\mathbf{x}_{\text{conv}}, \mathbf{x}_{\text{ac},1}, \mathbf{x}_{\text{ac},2}, \mathbf{x}_{\text{dc},1}, \mathbf{x}_{\text{dc},2})$ is a vector of all decision variables, where $\mathbf{x}_{\text{conv}} = (I_{di}, \theta_{ci}, \varphi_{ci}, k_{Ti}, i \in \mathbb{N}^{\text{conv}})$, $\mathbf{x}_{\text{ac},1} = (P_{Gi}, P_{ci}, Q_{Gi}, Q_{ci}, Q_{cci}, \dot{U}_i, i \in \mathbb{N}^{\text{ac}})$, $\mathbf{x}_{\text{ac},2} = (P_{ij}, Q_{ij}, \dot{I}_{ij}, (i, j) \in \mathbb{E}^{\text{ac}})$, $\mathbf{x}_{\text{dc},1} = (P_{di}, U_{di}, i \in \mathbb{N}^{\text{dc}})$ and $\mathbf{x}_{\text{dc},2} = (P_{dij}, I_{dij}, (i, j) \in \mathbb{E}^{\text{dc}})$. Let cardinalities be $|\mathbb{N}^{\text{ac}}| = n_{\text{ac}}$, $|\mathbb{N}^{\text{dc}}| = n_{\text{dc}}$, $|\mathbb{N}^{\text{conv}}| = n_{\text{c}}$, $|\mathbb{E}^{\text{ac}}| = m_{\text{ac}}$, and $|\mathbb{E}^{\text{dc}}| = m_{\text{dc}}$. Dimension of decision variable vector is $n_{\mathbf{x}} = 4n_{\text{c}} + 6n_{\text{ac}} + 2n_{\text{dc}} + 3m_{\text{ac}} + 2m_{\text{dc}}$.

3) Feasible Region

Feasible region $\mathbb{X}$ is described by constraints of the LCC station and AC-DC power grid model.

(1) Constraints of the LCC Station

Formulas of classical LCC station model (1)–(7) are adopted as constraints of the LCC station.

(2) Constraints of AC-DC Power Grid

Constraints of AC-DC power grid are formulated using BFM in [18]. For AC power grid, we have

$$P_{Gi} - P_{Li} - P_{ci} = \sum_{j:i \rightarrow j} P_{ij} - \sum_{k:k \rightarrow i} (P_{ki} - r_{ki} I_{ki}^2) \quad \forall i \in \mathbb{N}^{\text{ac}} \quad (20)$$

$$Q_{Gi} - Q_{Li} - Q_{ci} + Q_{cci} = \sum_{j:i \rightarrow j} Q_{ij} - \sum_{k:k \rightarrow i} (Q_{ki} - x_{ki} I_{ki}^2) \quad \forall i \in \mathbb{N}^{\text{ac}} \quad (21)$$

$$\dot{U}_i - \dot{U}_j = Z_{ij} \dot{I}_{ij} \quad \forall (i, j) \in \mathbb{E}^{\text{ac}} \quad (22)$$

$$P_{ij} + jQ_{ij} = \dot{U}_i \dot{I}_{ij}^* \quad \forall (i, j) \in \mathbb{E}^{\text{ac}} \quad (23)$$

For DC power grid, we have

$$P_{di} - P_{Ldi} = \sum_{j:i \rightarrow j} P_{dij} - \sum_{k:k \rightarrow i} (P_{dki} - r_{dki} I_{dki}^2) \quad \forall i \in \mathbb{N}^{\text{dc}} \quad (24)$$

$$U_{di} - U_{dj} = r_{dij} I_{dij} \quad \forall (i, j) \in \mathbb{E}^{\text{dc}} \quad (25)$$

$$P_{dij} = U_{di} I_{dij} \quad \forall (i, j) \in \mathbb{E}^{\text{dc}} \quad (26)$$

Equations (20), (21), and (24) describe balance of nodal active and reactive power. Equations (22) and (25) represent Ohm's Law for AC and DC lines, respectively. Equations (23) and (26) show correlations between branch power flows, voltages, and currents.

Limits are imposed on generator output active power P_{Gi} , generator output reactive power Q_{Gi} , and converter station reactive power compensation Q_{cci} :

$$\underline{P}_{Gi} \leq P_{Gi} \leq \bar{P}_{Gi} \quad \forall i \in \mathbb{N}^{\text{ac}} \quad (27)$$

$$\underline{Q}_{Gi} \leq Q_{Gi} \leq \bar{Q}_{Gi} \quad \forall i \in \mathbb{N}^{\text{ac}} \quad (28)$$

$$\underline{Q}_{cci} \leq Q_{cci} \leq \bar{Q}_{cci} \quad \forall i \in \mathbb{N}^{\text{ac}} \quad (29)$$

If bus i is not connected to a generator or a converter, let $\underline{P}_{Gi} = \bar{P}_{Gi} = \underline{Q}_{Gi} = \bar{Q}_{Gi} = 0$. If bus i is not connected to a converter, let $\underline{Q}_{cci} = \bar{Q}_{cci} = 0$.

Voltage magnitudes U_i and U_{di} are bounded by

$$\underline{v}_i \leq U_i^2 \leq \bar{v}_i \quad \forall i \in \mathbb{N}^{\text{ac}} \quad (30)$$

$$\underline{v}_{di} \leq U_{di}^2 \leq \bar{v}_{di} \quad \forall i \in \mathbb{N}^{\text{dc}} \quad (31)$$

Apparent power flowing through AC lines and active power flowing through DC lines P_{dij} are limited by

$$P_{ij}^2 + Q_{ij}^2 \leq \bar{S}_{ij}^2 \quad \forall (i, j) \in \mathbb{E}^{\text{ac}} \quad (32)$$

$$-\bar{P}_{dij} \leq P_{dij} \leq \bar{P}_{dij} \quad \forall (i, j) \in \mathbb{E}^{\text{dc}} \quad (33)$$

In summary, feasible region $\mathbb{X} \subseteq \mathbb{C}^{n_{\mathbf{x}}}$ is defined as

$$\mathbb{X} = \{\mathbf{x} \in \mathbb{C}^{n_{\mathbf{x}}} | \mathbf{x} \text{ satisfies (1)–(7), (20)–(33)}\} \quad (34)$$

Because (1)–(2), (4)–(5), (20)–(24), and (26) are nonlinear equalities, feasible region $\mathbb{X}$ is nonconvex, and the ADOL

described by (18) is a nonconvex optimization. In Section III, we will deduce an exact convex relaxation method to handle the ADOL efficiently.

B. Convexification of the ADOL

For constraints of the LCC station, formulas of the proposed LCC station model, (3), (10)–(12), and (15)–(17), are used to replace (1)–(7).

For constraints of AC-DC power grid, we apply the convexification method in [18] to reformulate (20)–(22) and (24) as affine (23) and (26) as quadratic equations. We define $v_i = U_i^2$, $v_{di} = U_{di}^2$, $\ell_{ij} = I_{ij}^2$, and $\ell_{dij} = I_{dij}^2$. Equations (20)–(23) in AC grid model are reformulated as

$$P_{Gi} - P_{Li} - P_{ci} = \sum_{j:i \rightarrow j} P_{ij} - \sum_{k:k \rightarrow i} (P_{ki} - r_{ki}\ell_{ki}) \quad \forall i \in \mathbb{N}^{\text{ac}} \quad (35)$$

$$Q_{Gi} - Q_{Li} - Q_{ci} + Q_{cci} = \sum_{j:i \rightarrow j} Q_{ij} - \sum_{k:k \rightarrow i} (Q_{ki} - x_{ki}\ell_{ki}) \quad \forall i \in \mathbb{N}^{\text{ac}} \quad (36)$$

$$v_i - v_j = 2(r_{ij}P_{ij} + x_{ij}Q_{ij}) - (r_{ij}^2 + x_{ij}^2)\ell_{ij} \quad \forall (i, j) \in \mathbb{E}^{\text{ac}} \quad (37)$$

$$P_{ij}^2 + Q_{ij}^2 = v_i\ell_{ij} \quad \forall (i, j) \in \mathbb{E}^{\text{ac}} \quad (38)$$

Equations (24)–(26) in DC grid model are reformulated as

$$P_{di} - P_{Ldi} = \sum_{j:i \rightarrow j} P_{dij} - \sum_{k:k \rightarrow i} (P_{dki} - r_{dki}\ell_{dki}) \quad \forall i \in \mathbb{N}^{\text{dc}} \quad (39)$$

$$v_{di} - v_{dj} = 2r_{dij}P_{dij} - r_{dij}^2\ell_{dij} \quad \forall (i, j) \in \mathbb{E}^{\text{dc}} \quad (40)$$

$$P_{dij}^2 = v_{di}\ell_{dij} \quad \forall (i, j) \in \mathbb{E}^{\text{dc}} \quad (41)$$

Because information of phase angles is lost after complex voltage and current variables are replaced by their magnitudes squared, we refer to model described by (3), (10)–(12), (15)–(17), (27)–(33), and (35)–(41) as *angle relaxation model*.

Let $\mathbf{y}_{\text{conv}} = (\ell_{di}, v'_{si}, v''_{si}, i \in \mathbb{N}^{\text{conv}})$, $\mathbf{y}_{\text{ac},1} = (P_{Gi}, P_{ci}, Q_{Gi}, Q_{ci}, Q_{cci}, v_i, i \in \mathbb{N}^{\text{ac}})$, $\mathbf{y}_{\text{ac},2} = (P_{ij}, Q_{ij}, \ell_{ij}, (i, j) \in \mathbb{E}^{\text{ac}})$, $\mathbf{y}_{\text{dc},1} = (P_{di}, v_{di}, i \in \mathbb{N}^{\text{dc}})$ and $\mathbf{y}_{\text{dc},2} = (P_{dij}, \ell_{dij}, (i, j) \in \mathbb{E}^{\text{dc}})$. A solution vector of angle relaxation model is denoted by $\mathbf{y} = (\mathbf{y}_{\text{conv}}, \mathbf{y}_{\text{ac},1}, \mathbf{y}_{\text{ac},2}, \mathbf{y}_{\text{dc},1}, \mathbf{y}_{\text{dc},2})$ with $n_{\mathbf{y}} = n_{\mathbf{x}} - n_{\mathbf{c}}$ real variables. Let $\mathbb{Y} \subseteq \mathbb{R}^{n_{\mathbf{y}}}$ be the set of all solutions of angle relaxation model:

$$\mathbb{Y} = \left\{ \mathbf{y} \in \mathbb{R}^{n_{\mathbf{y}}} \mid \mathbf{y} \text{ satisfies (3), (10)–(12), (15)–(17), (27)–(33), (35)–(41)} \right\} \quad (42)$$

ADOL with angle relaxation is formulated as follows:

$$\text{ADOL-ar:} \quad \min_{\mathbf{y}} \{f(\mathbf{y}) \mid \mathbf{y} \in \mathbb{Y}\} \quad (43)$$

Feasible region $\mathbb{Y}$ is still nonconvex due to quadratic equalities in (15), (16), (38) and (41). These equalities can be relaxed to rotated SOC constraints [18] as follows:

$$P_{di}^2 \leq v_{di}\ell_{di} \Leftrightarrow \left\| \begin{bmatrix} 2P_{di} \\ v_{di} - \ell_{di} \end{bmatrix} \right\|_2 \leq v_{di} + \ell_{di} \quad \forall i \in \mathbb{N}^{\text{conv}} \quad (44)$$

$$P_{ci}^2 + Q_{ci}^2 \leq v'_{si}\ell_{di} \Leftrightarrow \left\| \begin{bmatrix} 2P_{ci} \\ 2Q_{ci} \\ v'_{si} - \ell_{di} \end{bmatrix} \right\|_2 \leq v'_{si} + \ell_{di} \quad \forall i \in \mathbb{N}^{\text{conv}} \quad (45)$$

$$P_{ij}^2 + Q_{ij}^2 \leq v_i\ell_{ij} \Leftrightarrow \left\| \begin{bmatrix} 2P_{ij} \\ 2Q_{ij} \\ v_i - \ell_{ij} \end{bmatrix} \right\|_2 \leq v_i + \ell_{ij} \quad \forall (i, j) \in \mathbb{E}^{\text{ac}} \quad (46)$$

$$P_{dij}^2 \leq v_{di}\ell_{dij} \Leftrightarrow \left\| \begin{bmatrix} 2P_{dij} \\ v_{di} - \ell_{dij} \end{bmatrix} \right\|_2 \leq v_{di} + \ell_{dij} \quad \forall (i, j) \in \mathbb{E}^{\text{dc}} \quad (47)$$

We refer to model described by (3), (10)–(12), (17), (27)–(33), (35)–(37), (39)–(40), and (44)–(47) as *SOC relaxation model*. We define region $\bar{\mathbb{Y}} \subseteq \mathbb{R}^{n_{\mathbf{y}}}$:

$$\bar{\mathbb{Y}} = \left\{ \mathbf{y} \in \mathbb{R}^{n_{\mathbf{y}}} \mid \mathbf{y} \text{ satisfies (3), (10)–(12), (17), (27)–(33), (35)–(37), (39)–(40), (44)–(47)} \right\} \quad (48)$$

Note $\bar{\mathbb{Y}}$ is convex. Although $\text{sign}(P_{di})$ in (10) is nonconvex, direction of power injection from the LCC is designated, and $\text{sign}(P_{di})$ is typically a fixed value (−1 in inverter buses, 1 in rectifier buses and 0 in others). Hence, (10) is convex.

ADOL-ar with SOC relaxation is formulated as follows.

$$\text{ADOL-socr:} \quad \min_{\mathbf{y}} \{f(\mathbf{y}) \mid \mathbf{y} \in \bar{\mathbb{Y}}\} \quad (49)$$

We define projection $h: \mathbb{C}^{n_{\mathbf{x}}} \rightarrow \mathbb{R}^{n_{\mathbf{y}}}$ by $h(\mathbf{x}) = \mathbf{y}$, where

$$v'_{si} = k_{Ti}^2 U_i^2, \quad v''_{si} = k_{Ti}^2 U_i^2 \cos^2 \theta_{ci} \quad (50)$$

$$v_i = U_i^2, \quad v_{di} = U_{di}^2, \quad \ell_{ij} = I_{ij}^2, \quad \ell_{di} = I_{di}^2, \quad \ell_{dij} = I_{dij}^2 \quad (51)$$

and mappings of other elements in $\mathbf{y}$ are identity mappings. Here, $\bar{\mathbb{Y}} \supseteq \mathbb{Y} \supseteq h(\mathbb{X})$. Hence, ADOL-socr in (49) provides a lower bound for ADOL (18) and the ADOL-ar (43). If $\bar{\mathbb{Y}}$ and $f(\mathbf{y})$ are both convex, the ADOL-socr is a convex optimization that can be handled readily by a variety of solution techniques [46].

IV. EXACTNESS CONDITIONS AND SOLUTION RECOVERY

In this section, we discuss exactness of the ADOL-socr regarding the ADOL-ar considering different topological structures of DC grid. Exactness of the ADOL-ar regarding original ADOL was discussed in [18] and is not within the scope of this paper. For the ADOL-ar, the ADOL-socr is an exact relaxation if one of its optimal solutions attains equality in (44)–(47). We consider TTDC and MTDC separately according to different current shunts at converter bus. When there is no current shunt at converter bus, (44) is equivalent to (47), and there are milder sufficient conditions that guarantee exactness of relaxation.

A. Assumptions

Let $\ell = (\ell_{ij}, (i, j) \in \mathbb{E}^{\text{ac}})$ and $\ell_d = (\ell_{dij}, (i, j) \in \mathbb{E}^{\text{dc}})$ be current magnitudes squared in AC and DC branches, $\mathbf{S} = (P_{ij}, Q_{ij}, (i, j) \in \mathbb{E}^{\text{ac}})$ and $\mathbf{P}_d = (P_{dij}, (i, j) \in \mathbb{E}^{\text{dc}})$ be power flows in AC and DC branches, and $\mathbf{S}_L = (P_{Li}, Q_{Li}, i \in \mathbb{N}^{\text{ac}})$ and $\mathbf{P}_{Ld} = (P_{Ldi}, i \in \mathbb{N}^{\text{dc}})$ be power loads at AC and DC buses, respectively.

The following assumptions are given in this paper:

A1) The objective function $f(\mathbf{y})$ is convex.

A2) The objective function $f(\mathbf{y})$ is nondecreasing in ℓ and ℓ_d , nonincreasing in S_L and P_{Ld} , and independent of S and P_d .

A3) Loads S_L and P_{Ld} satisfy $\underline{S}_L \preceq S_L \preceq +\infty$ and $\underline{P}_{Ld} \preceq P_{Ld} \preceq +\infty$, respectively, where $\underline{S}_L$ and $\underline{P}_{Ld}$ are given loads.

Assumptions A1–A3 are originally sufficient conditions of exact SOCP relaxation for AC OPF provided by [18]. They are further extended in this paper to sufficient conditions of exact relaxation for SOC constraints (46)–(47) of AC and DC grids in the ADOL. We use a milder condition—“nondecreasing” in assumption A2—than the condition “strictly increasing” in [18] to guarantee exact relaxation of (46)–(47). This change may lead to optimal solution of relaxed model attaining strict inequality in (46)–(47) when $f(\mathbf{y})$ is not strictly increasing in ℓ and ℓ_d ; however, an optimal solution that attains equality in (46)–(47) can be readily constructed from it. When constructing such a solution, procedure for dealing with (46)–(47) can be found in [26].

In what follows, we focus on exactness of converter-side SOC constraints (44)–(45) and provide sufficient conditions along with their proofs.

B. Exactness of Relaxation for the ADOL-socr with TTDC

A TTDC system is a DC grid with only one rectifier station and one inverter station connected by one DC line. If P_{Ldi} can be omitted, we have

$$P_{di} = P_{dij} \quad \forall i \in \mathbb{N}^{\text{conv}} \quad (52)$$

$$\ell_{di} = \ell_{dij} \quad \forall i \in \mathbb{N}^{\text{conv}} \quad (53)$$

In this case, (44) is equivalent to (47). Equation (53) can be added in the ADOL-socr to guarantee the optimal solution satisfies equality of (44). To prove the proposed relaxation is exact, it suffices to show optimal solution of the ADOL-socr attains equality in (45). Here, we provide sufficient conditions of exact relaxation for (45) in Proposition 1.

Proposition 1. Suppose f is not explicitly dependent on Q_{ci} and Q_{cci} for $\forall i \in \mathbb{N}^{\text{conv}}$. If $\overline{Q}_{cci} = +\infty$ for $\forall i \in \mathbb{N}^{\text{conv}}$ there exists an optimal solution of the ADOL-socr that attains equality in (45).

Proof of Proposition 1 is provided in Appendix A. We briefly outline the proof here. In proof of Proposition 1, we first assume $\mathbf{y}_*$ is the optimal solution of the ADOL-socr and construct from $\mathbf{y}_*$ a solution $\tilde{\mathbf{y}}$ with all SOC constraints active. Next, we validate $\tilde{\mathbf{y}}$ is not only a feasible solution of the ADOL-socr and ADOL-ar but also an optimal solution of the ADOL-socr. Since optimal value of the ADOL-ar is not smaller than ADOL-socr, $\tilde{\mathbf{y}}$ is also the optimal solution of ADOL-ar.

Remark 1. We refer to $\tilde{\mathbf{y}}$ constructed from $\mathbf{y}_*$ as *constructed solution*. Proposition 1 illustrates the optimal solution of the ADOL-socr may attain strict inequality in (45), i.e., may be infeasible for the ADOL-ar. However, under certain conditions, there is another optimal point of the ADOL-socr that attains equality in (45), which is indeed optimal for the ADOL-ar. Proof of Proposition 1 provides conditions and method of

finding such an optimal solution. Hence, the ADOL-socr is exact for TTDC.

Remark 2. In the ADOL-socr, Q_{ci} can be regarded as an adjustable reactive power load at bus $i \in \mathbb{N}^{\text{conv}}$. RPCDs installed at the same bus $i \in \mathbb{N}^{\text{conv}}$ meet reactive power demand of AC grid while minimizing transmission losses of nearby lines before they inject reactive power into converter to reach optimal level. Hence, for $\forall i \in \mathbb{N}^{\text{conv}}$, although Proposition 1 requires $\overline{Q}_{cci} = +\infty$, actual upper bound of Q_{cci} does not exceed need for achieving a local reactive power balance. Additionally, there may exist a mild condition, but it is challenging to determine an accurate bound before solving the problem such that the relaxed model is exact because this bound is determined by optimal values of decision variables. There may be some way to obtain an accurate bound based fully on the model parameters, which warrants investigation in the future.

C. Exactness for the ADOL-socr with MTDC

An MTDC system consists of a DC power grid with three or more converter stations. At least one converter station is connected to two or more converter stations. Equation (53) does not hold in MTDC. More conditions are needed to ensure the optimal solution attains equality in (44) and (45). Here, we provide sufficient conditions of exact relaxation for (44) and (45) in Proposition 2.

Proposition 2. Suppose f is independent of ℓ_{di} , v'_{si} , v''_{si} , Q_{ci} and Q_{cci} for $\forall i \in \mathbb{N}^{\text{conv}}$. If $v_{di} \geq k_{Ti}^2 \cdot \cos^2 \theta_{ci} \cdot \bar{v}_i$ and $\overline{Q}_{cci} = +\infty$ for $\forall i \in \mathbb{N}^{\text{conv}}$ there exists an optimal solution of the ADOL-socr that satisfies equality of (44) and (45).

Proof of Proposition 2 is provided in Appendix B. It is similar to proof of Proposition 1.

Remark 3. Proposition 2 provides sufficient conditions to ensure an optimal solution of the ADOL-socr satisfying equality of (44) and (45) can be constructed from any other optimal solution of ADOL-socr. Hence, the ADOL-socr for MTDC is also exact.

Remark 4. We ignore (44) in TTDC since we have (52)–(53) alternatively based on the condition that P_{Ldi} can be omitted for $\forall i \in \mathbb{N}^{\text{conv}}$. If this condition does not hold, equality of (44) still needs to be considered, which means the exact convexification of the ADOL-socr with TTDC should also satisfy Proposition 2 instead of Proposition 1.

D. Discussion of the Effect of Relaxation

In this subsection, we discuss effect of relaxation on the model to make conditions and proofs given by Propositions 1 and 2 easier to understand.

The effect of relaxing (44) is analyzed as follows. Because Q_{ci} is equivalent to a reactive power load in (36), a smaller Q_{ci} leads to lower transmission losses if there are no excess reactive power injections near converter bus. Q_{cci} is equivalent to a reactive power source in (36), which can supply reactive power demand at converter bus without additional costs. Therefore, Q_{ci} in relaxed model can be compensated by Q_{cci} at no cost; thus, relaxed model has multiple optimal solutions with varying Q_{ci} and Q_{cci} . Whether there is an

optimal solution satisfying equality in (44) or not is determined by value range of Q_{cci} , and conditions claimed in Proposition 1 guarantee uniqueness of optimal solution.

Effect of relaxing (45) is next discussed. After relaxing (45), injected current ℓ_{di} at converter bus is not directly related to DC transmission current ℓ_{dij} flowing out of converter bus; thus, it is not related to transmission losses of DC line or cost. By varying ℓ_{di} , values of the converter transformer ratio k_{Ti} and converter control angle θ_{ci} can be changed accordingly, by which multiple optimal solutions to relaxed model can be constructed. If bounds of k_{Ti} and θ_{ci} satisfy conditions in Proposition 2, there exists a unique optimal solution attaining equality in (45).

Because sufficient conditions provided in Propositions 1 and 2 do not contradict any condition required in A1–A3, equality in (44)–(47) can all be attained in the ADOL-socr.

E. Solution Recovery

Proofs of Propositions 1 and 2 show how to construct an optimal solution for the ADOL-ar from the ADOL-socr. Since there is a projection mapping $h : \mathbb{C}^{n_x} \rightarrow \mathbb{R}^{n_y}$ between original solution $\mathbf{x}$ and relaxed solution $\mathbf{y}$, we need an inverse projection to map a solution $\mathbf{y}$ of the ADOL-ar to a feasible solution $\mathbf{x}$ of original ADOL. For each $\theta = (\theta_i, i \in \mathbb{N}^{ac}) \in (-\pi, \pi]^{n_{ac}}$, we define inverse projection $h_\theta : \mathbb{R}^{n_y} \rightarrow \mathbb{C}^{n_x}$ by $h_\theta(\mathbf{y}) = \mathbf{x}$, where

$$k_{Ti} = \sqrt{\frac{v'_{si}}{v_i}}, \theta_{ci} = \arccos \sqrt{\frac{v''_{si}}{v'_{si}}}, \varphi_{ci} = \arccos \sqrt{\frac{v_{di}}{v'_{si}}} \quad (54)$$

$$\dot{U}_i = \sqrt{v_i} e^{j\theta_i}, \dot{I}_{ij} = \sqrt{\ell_{ij}} e^{j[\theta_i - \angle(P_{ij} + jQ_{ij})]}, U_{di} = \sqrt{v_{di}} \quad (55)$$

$$I_{di} = \sqrt{\ell_{di}} \cdot \text{sign}(P_{di}), I_{dij} = \sqrt{\ell_{dij}} \cdot \text{sign}(P_{dij}) \quad (56)$$

and mapping relationships of the other elements in $\mathbf{x}$ are identity mappings. Here, θ refers to phase angles of AC bus voltages. Hence, whether we can recover a solution of ADOL from a relaxed solution $\mathbf{y}$ depending on whether there exists θ such that inverse projection $h_\theta(\mathbf{y})$ lies in $\mathbb{X}$.

Reference [18] provides the unique existence condition of θ . This condition indicates in a radial network, phase angles of any relaxed solution $\mathbf{y}$ can be recovered via angle recovery method in [18]. In a mesh network, unique existence condition may not held because relaxed cycle condition for phase angles may not be satisfied by relaxed solution, except when phase shifters are installed in every cycle of the network [47].

V. SIMULATION RESULTS

Four test cases are conducted to illustrate properties and performance of the proposed model. A 12-bus test system with embedded MTDC is employed to validate the proposed LCC station model in Case I, numerically examine the proposed propositions in Case II, and verify effectiveness of the proposed solution construction methods in Case III. Furthermore, in Case IV, efficiency of the ADOL-socr is compared with the ADOL-ar in four test systems of different scales.

Exactness of the ADOL-socr is evaluated by calculating gaps between the two sides of SOC (44)–(47):

$$\varepsilon_{ci} = (v'_{si}\ell_{di} - P_{ci}^2 - Q_{ci}^2)/v'_{si}\ell_{di} \quad i \in \mathbb{N}^{\text{conv}} \quad (57)$$

$$\varepsilon_{di} = (v_{di}\ell_{di} - P_{di}^2)/v_{di}\ell_{di} \quad i \in \mathbb{N}^{\text{conv}} \quad (58)$$

$$\varepsilon_{ij} = (v_i\ell_{ij} - P_{ij}^2 - Q_{ij}^2)/v_i\ell_{ij} \quad (i, j) \in \mathbb{E}^{\text{ac}} \quad (59)$$

$$\varepsilon_{dij} = (v_{di}\ell_{dij} - P_{dij}^2)/v_{di}\ell_{dij} \quad (i, j) \in \mathbb{E}^{\text{dc}} \quad (60)$$

Values of these gaps lie within $[0, 1]$. The ADOL-socr is considered to be exact if these gaps are all smaller than a predefined threshold ε , which is set to 10^{-3} in this paper.

All simulations are performed on a computer with six processors running at 2.90 GHz with 16 GB of memory. The solution of the OPFs is coded in MATLAB R2017a and solved using the GUROBI and IPOPT solvers.

A. Case I: Validation of the Proposed LCC Station Model

In Case I, results obtained by solving the ADOL and the ADOL-ar based on a 12-bus system are used to validate the proposed LCC station model. 12-bus system is shown in Fig. 2. Buses 3 and 12 are connected to two rectifier stations, and buses 6 and 9 are connected to inverter stations. Upper limits of active generation outputs at buses 1, 4, 7, and 10 are set to 1400 MW, 300 MW, 300 MW, and 1000 MW, respectively. Active power loads at buses 2, 5, 8, and 11 are set to 400 MW, 700 MW, 800 MW, and 500 MW, respectively. Impedances of all AC lines are $0.001 + j0.01$ p.u., and resistances of all HVDC lines are 0.001 p.u. Converter–transformer ratio is restricted to between 0.9 and 1.1. Control angle of rectifier is restricted to between 12.5° and 17.5° . Control angles of the inverter are between 15.5° and 20.5° . We designed the 12-bus test system for theoretical studies instead of conducting actual testing. Generation cost coefficients for the 12-bus test system can be taken arbitrarily because they do not influence conclusions obtained from any simulations in this section. In this paper, quadratic, linear and constant cost coefficients are taken as 0.01, 0.3 and 0.2, respectively.

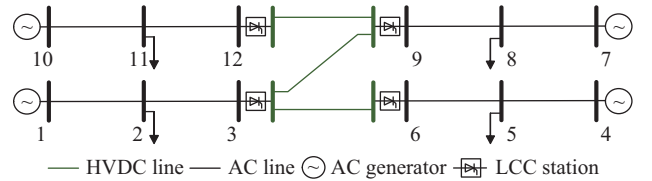


Fig. 2. The 12-bus test system with embedded MTDC.

We show in Table I maximum relative error (MRE) and average relative error (ARE) of variables of the LCC stations obtained by solving the ADOL and ADOL-ar. These results are negligibly small, implying the proposed LCC station model is fully equivalent to the classical LCC station model.

B. Case II: Numerical Examination of Propositions

In Case II, we examine Propositions 1 and 2 numerically via two tests using the 12-bus system. We aim to visualize discussions in Section IV-D to intuitively explain why optimal solutions of the relaxed model ADOL-socr contain those of the ADOL-ar.

1) Test 1: Properties of Current Injection to Converters

Due to relaxation in (44), ℓ_{di} may be larger than sum of currents ℓ_{dij} of all branches connected to bus i , resulting in

TABLE I
VARIABLE ERRORS OF THE OPTIMAL SOLUTIONS OF THE ADOL AND ADOL-AR

	MRE (%)	ARE (%)		MRE (%)	ARE (%)
P_{ci}	7.14×10^{-5}	3.21×10^{-5}	I_{di}	1.47×10^{-4}	6.65×10^{-5}
Q_{ci}	8.70×10^{-5}	3.64×10^{-5}	θ_{ci}	9.94×10^{-5}	3.69×10^{-5}
Q_{cci}	7.48×10^{-4}	3.19×10^{-4}	φ_{ci}	1.57×10^{-5}	1.17×10^{-5}
U_{di}	5.43×10^{-6}	5.02×10^{-6}	k_{Ti}	4.21×10^{-5}	1.36×10^{-5}

nonzero gaps in corresponding SOC constraints. On the other hand, ℓ_{di} is constrained in (10), which leads to a voltage drop in commutation. In this test, relationship of ℓ_{di} versus generation cost and gap of (44) is investigated.

The ADOL-socr is solved with $\ell_{d,3}$ increased from its zero-gap optimal value to observe variation in generation cost and gaps of (44). As shown in Figs. 3 and 4, there are two stages in solely changing $\ell_{d,3}$. In Stage A, cost remains unchanged because $v''_{s,3}$ can be adjusted by varying control angle $\theta_{c,3}$ or converter transformer ratio $k_{T,3}$ to adapt to change in $\ell_{d,3}$. In addition, $k_{T,3}$ remains unchanged in Fig. 4 since it is at its upper limit at initial point. In Stage B, after $\theta_{c,3}$ and $k_{T,3}$ reach their limits, increased $\ell_{d,3}$ leads to a larger commutation voltage drop. Consequently, transmission losses on DC lines and cost also increase accordingly.

In summary, a nonzero ε_{di} for any bus i in MTDC in optimal solution can be pulled to zero by reducing v'_{si} or v''_{si} (k_{Ti} or θ_{ci}) because conditions in Proposition 2 guarantee v'_{si} and v''_{si} (k_{Ti} or θ_{ci}) do not reach their boundaries until ε_{di} is zero.

2) Test 2: Properties of Reactive Power

In this test, we focus on properties of reactive power injection into LCC (Q_{ci}) and reactive compensation (Q_{cci}), which are key variables in Propositions 1 and 2.

The ADOL-socr is solved with a specified Q_{cci} varying from 0 to 500 Mvar at each converter-connected buses (i.e., buses 3 and 12) to explore its effect on cost and ε_{ci} . Each of the curves in Fig. 5 can be divided into three stages.

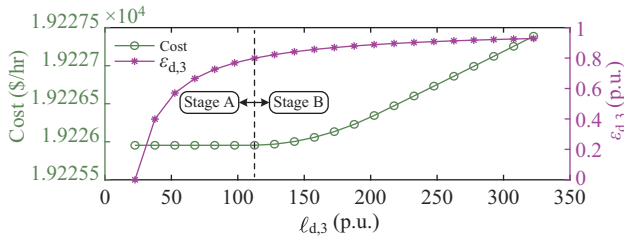


Fig. 3. Variation in cost and ε_{di} versus ℓ_{di} at bus 3.

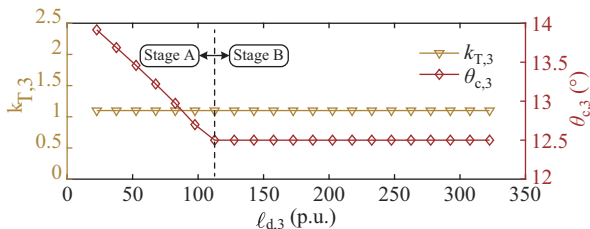


Fig. 4. Variation in k_{Ti} and θ_{ci} versus ℓ_{di} at bus 3.

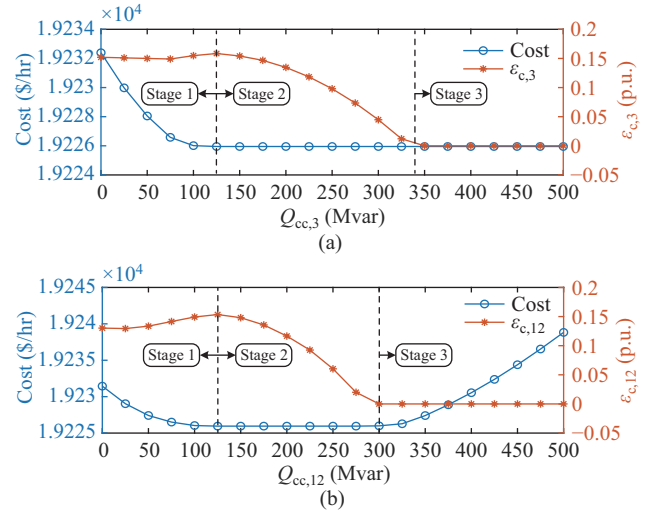


Fig. 5. Variation in cost and ε_{ci} versus Q_{cci} at buses 3 and 12. (a) Bus 3. (b) Bus 12.

In Stage 1, all reactive power compensation is provided to AC grid to achieve a local reactive power balance and reduce transmission losses, which decrease total cost.

In Stage 2, nodal reactive power balance reaches optimal level, and the rest of reactive power compensation is injected into the LCC. Hence, ε_{ci} decreases, and cost remains unchanged.

In Stage 3, ε_{ci} reaches zero, and excessive reactive power compensation must be consumed by AC grid or the LCC. If excessive reactive power is consumed by AC grid, transmission losses will increase because nodal reactive power balance will be farther from its optimal level in Stage 2. If excessive reactive power is consumed by the LCC, the ℓ_{dij} in TTDC will increase since $\ell_{di} = \ell_{dij}$. Therefore, cost increases, and ε_{ci} remains 0 in the test of bus 12, which is equivalent to converter buses of TTDC.

Results for bus 3 are slightly different from those for bus 12 because bus 3 is connected to a converter of MTDC. For bus 3, ℓ_{di} is in Stage A when Q_{cci} is in Stage 3. Hence, cost remains unchanged and does not increase.

To conclude, every point in Stage 2 is an optimal point of the ADOL-socr. In particular, for the ADOL-socr with TTDC, if an optimal point is such that Q_{cci} for $\forall i \in \mathbb{N}^{\text{conv}}$ is on the boundary between Stages 2 and 3, it is an exact solution that has not only minimum cost but also zero gap in (45). This boundary exists as long as upper limits on Q_{cci} are sufficiently large, which is guaranteed by condition in Proposition 1.

C. Case III: Verification of Solution Construction

In Case III, proposed solution construction method is tested using the 12-bus system. We solve the ADOL-socr to obtain a primitive optimal solution (POS) and calculate its gaps for SOC constraints, as shown in Table II. For simplicity, gaps less than $\varepsilon = 10^{-3}$ are considered to be sufficiently small.

POS is the optimal solution obtained by solving ADOL-socr directly. We observe ε_{ci} is nonzero at all converter buses for POS because Q_{ci} can be regarded as slack reactive power extraction after relaxation. This variable is compensated only

TABLE II
GAPS OF THE SOC CONSTRAINTS AND COST IN THE POS AND COS

Bus	ε_{ci} (p.u.)		ε_{di} (p.u.)	
	POS	COS	POS	COS
3	0.368	3.52×10^{-16}	0.698	3.17×10^{-13}
12	0.076	2.14×10^{-16}	2.39×10^{-9}	2.39×10^{-9}
6	0.115	2.55×10^{-16}	1.04×10^{-12}	1.04×10^{-12}
9	0.070	1.16×10^{-16}	4.87×10^{-12}	4.87×10^{-12}
Maximum	ε_{ij} (p.u.)		ε_{dij} (p.u.)	
	POS	COS	POS	COS
	1.78×10^{-8}	1.78×10^{-8}	1.85×10^{-8}	1.85×10^{-8}
Cost (\$/hr)	POS		COS	
	19225.95		19225.95	

by Q_{cci} if there is no excess reactive power at bus i . In addition, ε_{di} is zero at all buses except bus 3. Buses 6 and 12 can be regarded as converter buses of TTDC, in which case (44) is equivalent to (47), and we have $\varepsilon_{d,6} = \varepsilon_{d,3,6} < \varepsilon$ and $\varepsilon_{d,12} = \varepsilon_{d,9,12} < \varepsilon$. For bus 3, $\ell_{d,3}$ can be larger than sum of branch currents caused by the relaxation, the change in which can be compensated by varying $\theta_{c,3}$ or $k_{T,3}$, as analyzed in Case II. For bus 9, $\theta_{c,9}$ and $k_{T,9}$ have already reached their limits for optimal operating point, and $\ell_{d,9}$ cannot be increased without cost at bus 3; hence, $\varepsilon_{d,9} < \varepsilon$. According to Section IV-D, the ADOL-socr has multiple optimal solutions due to relaxation. Gaps of SOC constraints (44)–(45) in these optimal solutions are nonzero except for exact optimal solution. Therefore, there are almost always gaps in (44)–(45) in the POS.

Next, we obtain a constructed optimal solution (COS) by assigning corresponding values to variables that need to be changed in proofs of Propositions 1 and 2, including ℓ_{di} , v'_{si} , v''_{si} , Q_{ci} , Q_{cci} and Q_{Li} for $\forall i \in \mathbb{N}^{\text{conv}}$. Then, we calculate gaps of all SOC constraints and objective function value of the SOC to check whether COS is optimal for the ADOL-ar and ADOL-socr. As shown in Table II, ε_{ci} and ε_{di} are zero for $\forall i \in \mathbb{N}^{\text{conv}}$ for COS, indicating COS is feasible for the ADOL-ar. In addition, objective value of COS is equal to that of POS. It is validated COS is an optimal solution of the ADOL-ar.

In summary, as long as the ADOL-ar satisfies conditions in Propositions 1 and 2, all optimal solutions of relaxed model correspond to the same objective value as optimal solution of the ADOL-ar, and one of them attains equality in (44)–(45).

D. Case IV: Exactness and Computational Efficiency

In Case IV, results obtained by solving the 12-bus, 39-bus, 118-bus, and 300-bus systems are used to compare computational efficiency of the ADOL-socr with that of ADOL-ar. As a nonconvex optimization, the ADOL-ar is solved by IPOPT, which is based on interior point method [48]. The 39-bus system is modified by embedding 2 TTDC and 2 MTDC systems. The 118-bus and 300-bus systems are connected to another mirror system with the same data via 10 TTDCs and 5 MTDCs and 17 TTDCs and 8 MTDCs, respectively. Detailed data can be found in [49].

Results are given in Table III, in which solution of the ADOL-socr is referred to as constructed solution. Second and third rows show relative errors of optimal values between

TABLE III
ERRORS AND GAPS OF THE OPTIMAL SOLUTIONS AND RUNTIMES OF THE ADOL-SOCR AND ADOL-AR

Test System	12-bus	39-bus	118-bus	300-bus
Relative Error of Cost (%)	9.2×10^{-7}	5.4×10^{-6}	1.0×10^{-6}	1.2×10^{-4}
Maximum Gap	1.9×10^{-8}	3.3×10^{-6}	1.1×10^{-5}	1.4×10^{-5}
Variables MRE (%)	0.0623	0.9120	5.6146	0.6960
Variables ARE (%)	0.0025	0.0373	0.0529	0.0026
Runtime (s)	ADOL-socr 0.01	0.03	0.16	0.59
	ADOL-ar 0.48	1.05	36.43	1085.81

the ADOL-socr and ADOL-ar and maximum gaps of the SOC constraints of the ADOL-socr. Fourth and fifth rows are MRE and ARE of optimal solutions between the two models. Large relative errors are caused by near-zero values of some variables relating to power flows of transmission lines and generation outputs. All errors between optimal solutions of the two models are negligibly small, indicating exactness of the ADOL-socr is not significantly affected by scale of AC grid and numbers of TTDCs and MTDCs.

Sixth and seventh rows show total runtimes for solving the ADOL-socr and ADOL-ar, respectively. As a convex optimization, locally optimal solution of the ADOL-socr is also globally optimal. In contrast, the ADOL-ar is nonconvex, and there may exist multiple locally optimal solutions that prevent a global search by IPOPT solver. Hence, the ADOL-socr can be solved much more efficiently and effectively than the nonconvex problem.

As shown in Table III, relative errors of cost between the ADOL-socr and ADOL-ar and maximum gaps of the SOC constraints of the ADOL-socr in the four test systems are small enough to conclude the optimal solution of the ADOL-socr is also that of the ADOL-ar, which also indicates the ADOL-socr is exact for all four test systems.

VI. CONCLUSION

In this paper, we propose a new LCC station model with only linear formulas and quadratic equations and a convexification method for the ADOL. SOCP relaxation is used in quadratic equations in the ADOL. Sufficient conditions for exactness of the proposed ADOL-socr are provided and proven for TTDC and MTDC.

In simulations, we validated the proposed and classical LCC station models are exactly equivalent in test system. We also investigated properties of the SOC constraints from a physical perspective and numerically examined proposed propositions. In addition, our numerical simulation results demonstrate the proposed relaxed OPF enables us to obtain a globally optimal solution of original OPF. Moreover, we confirmed exactness and efficiency of the proposed relaxed model in test systems of different scales.

Based on the proposed ADOL-socr, several directions for improvement are open for future study. Obtaining milder conditions for exactness of the ADOL-socr would be noteworthy. Strengthening relaxation to obtain a tighter convex ADOL model is also an interesting issue that needs further study. Another important but challenging topic is to formulate a

convex AC-DC OPF with hybrid converters, such as LCC and VSC. All these issues merit further investigation.

APPENDIX A

Proof of Proposition 1: Let $\mathbf{y}_*$ be an optimal solution of the ADOL-socr. Consider another point $\tilde{\mathbf{y}}$ that satisfies

$$\tilde{Q}_{ci} = \sqrt{v'_{si*} \ell_{di*} - P_{ci*}^2} \quad i \in \mathbb{N}^{\text{conv}} \quad (\text{A1})$$

$$\tilde{Q}_{cci} = Q_{cci*} + \tilde{Q}_{ci} - Q_{ci*} \quad i \in \mathbb{N}^{\text{conv}} \quad (\text{A2})$$

Values of other variables in $\tilde{\mathbf{y}}$ are the same as in $\mathbf{y}_*$. To prove $\tilde{\mathbf{y}}$ is a feasible point, it suffices to check whether $\tilde{\mathbf{y}}$ satisfies (29), (36), and (45). For $\forall i \in \mathbb{N}^{\text{conv}}$, we obtain

$$\tilde{Q}_{cci} \geq Q_{cci*} \geq 0 \quad (\text{A3})$$

$$\begin{aligned} \tilde{Q}_{Gi} - \tilde{Q}_{Li} - \tilde{Q}_{ci} + \tilde{Q}_{cci} \\ = Q_{Gi*} - Q_{Li*} - \tilde{Q}_{ci} + (Q_{cci*} + \tilde{Q}_{ci} - Q_{ci*}) \\ = \sum_{j:i \rightarrow j} Q_{ij*} - \sum_{k:k \rightarrow i} (Q_{ki*} - x_{ki} \ell_{ki*}) \\ = \sum_{j:i \rightarrow j} \tilde{Q}_{ij} - \sum_{k:k \rightarrow i} (\tilde{Q}_{ki} - x_{ki} \tilde{\ell}_{ki}), \end{aligned} \quad (\text{A4})$$

$$\begin{aligned} \tilde{P}_{ci}^2 + \tilde{Q}_{ci}^2 &\leq P_{ci*}^2 + (v'_{si*} \ell_{di*} - P_{ci*}^2) \\ &\leq v'_{si*} \ell_{di*} \\ &= \tilde{v}'_{si} \tilde{\ell}_{di}. \end{aligned} \quad (\text{A5})$$

Hence, $\tilde{\mathbf{y}}$ satisfies (29), (36), and (45), and it is a feasible point for the ADOL-socr. Since f is independent of Q_{ci} and Q_{cci} for $\forall i \in \mathbb{N}^{\text{conv}}$, $\tilde{\mathbf{y}}$ yields the same objective value as $\mathbf{y}_*$. Therefore, $\tilde{\mathbf{y}}$ is also an optimal solution of the ADOL-socr. Since $\tilde{\mathbf{y}}$ is an optimal solution that attains equality in (45), it is also an optimal solution of the ADOL-ar.

This completes the proof.

APPENDIX B

Proof of Proposition 2: Let $\mathbf{y}_*$ be an optimal solution of the ADOL-socr. Consider another point $\tilde{\mathbf{y}}$ that satisfies

$$\tilde{\ell}_{di} = \frac{P_{di*}^2}{v_{di*}} \quad i \in \mathbb{N}^{\text{conv}} \quad (\text{B1})$$

$$\tilde{v}''_{si} = v''_{si*} - x_{Li}^2 (\ell_{di*} - \tilde{\ell}_{di}) \quad i \in \mathbb{N}^{\text{conv}} \quad (\text{B2})$$

$$\tilde{v}'_{si} = \frac{\tilde{v}''_{si}}{\max \left\{ \frac{\tilde{v}''_{si}}{v'_{si*}}, \cos^2 \theta_{ci} \right\}} \quad i \in \mathbb{N}^{\text{conv}} \quad (\text{B3})$$

$$\tilde{Q}_{ci} = \sqrt{|\tilde{v}'_{si} \tilde{\ell}_{di} - P_{ci*}^2|} \quad i \in \mathbb{N}^{\text{conv}} \quad (\text{B4})$$

$$\tilde{Q}_{cci} = \max\{Q_{cci*} + \tilde{Q}_{ci} - Q_{ci*}, 0\} \quad i \in \mathbb{N}^{\text{conv}} \quad (\text{B5})$$

$$\tilde{Q}_{Li} = \max\{Q_{Li*}, Q_{Li*} - Q_{cci*} - \tilde{Q}_{ci} + Q_{ci*}\} \quad i \in \mathbb{N}^{\text{conv}} \quad (\text{B6})$$

Values of other variables in $\tilde{\mathbf{y}}$ are the same as those in $\mathbf{y}_*$. To prove $\tilde{\mathbf{y}}$ is a feasible point, it suffices to verify $\tilde{\mathbf{y}}$ satisfies (10)–(12), (29), (36), (44), and (45).

Starting from the left side of (10) for $\tilde{\mathbf{y}}$, we obtain

$$\begin{aligned} \tilde{v}''_{si} - \tilde{v}_{di} \\ = [v''_{si*} - x_{Li}^2 (\ell_{di*} - \tilde{\ell}_{di})] - v_{di*} \end{aligned}$$

$$\begin{aligned} &= 2x_{Li} P_{di*} \cdot \text{sign}(P_{di*}) + x_{Li}^2 \ell_{di*} - x_{Li}^2 (\ell_{di*} - \tilde{\ell}_{di}) \\ &= 2x_{Li} \tilde{P}_{di} \cdot \text{sign}(\tilde{P}_{di}) + x_{Li}^2 \tilde{\ell}_{di} \end{aligned} \quad (\text{B7})$$

Hence, $\tilde{\mathbf{y}}$ satisfies (10).

There are two possible cases for $\tilde{v}'_{si}$, $\forall i \in \mathbb{N}^{\text{conv}}$:

1) If $\tilde{v}''_{si}/v'_{si*} \geq \cos^2 \theta_{ci}$, then $\tilde{v}'_{si} = v'_{si*}$ along with

$$\tilde{v}'_{si} \cos^2 \theta_{ci} = v'_{si*} \cos^2 \theta_{ci} \leq \tilde{v}''_{si} \quad (\text{B8})$$

$$\tilde{v}'_{si} \cos^2 \theta_{ci} = v'_{si*} \cos^2 \theta_{ci} \geq v''_{si*} > \tilde{v}''_{si}, \quad (\text{B9})$$

and

$$\underline{k}_{Ti}^2 \tilde{v}_i \leq \underline{k}_{Ti}^2 v_{i*} \leq v'_{si*} = \tilde{v}'_{si} \leq \bar{k}_{Ti}^2 v_{i*} = \bar{k}_{Ti}^2 \tilde{v}_i \quad (\text{B10})$$

From (B7) and (B9), we obtain

$$\tilde{v}'_{si} \cos^2 \theta_{ci} > \tilde{v}''_{si} = \tilde{v}_{di} + 2x_{Li} \tilde{P}_{di} \cdot \text{sign}(\tilde{P}_{di}) + x_{Li}^2 \tilde{\ell}_{di} > \tilde{v}_{di} \quad (\text{B11})$$

From (B1) and (B11), we have

$$\tilde{v}'_{si} \tilde{\ell}_{di} > \tilde{v}'_{si} \cos^2 \theta_{ci} \cdot \tilde{\ell}_{di} > \tilde{v}_{di} \tilde{\ell}_{di} = \tilde{P}_{di}^2 = \tilde{P}_{ci}^2 \quad (\text{B12})$$

Starting from the left side of (45) for $\tilde{\mathbf{y}}$, we have

$$\tilde{P}_{ci}^2 + \tilde{Q}_{ci}^2 = P_{ci*}^2 + (\tilde{v}'_{si} \tilde{\ell}_{di} - P_{ci*}^2) = \tilde{v}'_{si} \tilde{\ell}_{di} \quad (\text{B13})$$

2) If $\tilde{v}''_{si}/v'_{si*} < \cos^2 \theta_{ci}$, then from (B3), we have

$$\tilde{v}'_{si} \cos^2 \theta_{ci} = \tilde{v}''_{si} \quad (\text{B14})$$

Additionally, we have

$$\tilde{v}'_{si} \cos^2 \theta_{ci} \geq \tilde{v}'_{si} \cos^2 \theta_{ci} = \tilde{v}''_{si} \quad (\text{B15})$$

From (B14) and $\underline{v}_{di} \geq \underline{k}_{Ti}^2 \cdot \cos^2 \theta_{ci} \cdot \bar{v}_i$, we obtain

$$\underline{k}_{Ti}^2 \tilde{v}_i \leq \underline{k}_{Ti}^2 \bar{v}_i \leq \underline{v}_{di} / \cos^2 \theta_{ci} \leq \tilde{v}_{di} / \cos^2 \theta_{ci} < \tilde{v}'_{si} \quad (\text{B16})$$

and

$$\bar{k}_{Ti}^2 \tilde{v}_i = \bar{k}_{Ti}^2 v_{i*} \geq v'_{si*} > \tilde{v}''_{si} / \cos^2 \theta_{ci} = \tilde{v}'_{si} \quad (\text{B17})$$

In this case, (B11)–(B13) still hold. Hence, $\tilde{\mathbf{y}}$ satisfies (11), (12), and (45) in both cases.

For $\tilde{Q}_{cci}$ and $\tilde{Q}_{Li}$, $\forall i \in \mathbb{N}^{\text{conv}}$, there are two possible cases:

1) If $Q_{cci*} + \tilde{Q}_{ci} - Q_{ci*} > 0$, then $Q_{Li*} - Q_{cci*} - \tilde{Q}_{ci} + Q_{ci*} < Q_{Li*}$. According to (B5) and (B6), we have $\tilde{Q}_{cci} = Q_{cci*} + \tilde{Q}_{ci} - Q_{ci*} > 0$ and $\tilde{Q}_{Li} = Q_{Li*}$. Starting from the left side of (36) for $\tilde{\mathbf{y}}$, we have

$$\begin{aligned} \tilde{Q}_{Gi} - \tilde{Q}_{Li} - \tilde{Q}_{ci} + \tilde{Q}_{cci} \\ = Q_{Gi*} - Q_{Li*} - \tilde{Q}_{ci} + (Q_{cci*} + \tilde{Q}_{ci} - Q_{ci*}) \\ = \sum_{j:i \rightarrow j} Q_{ij*} - \sum_{k:k \rightarrow i} (Q_{ki*} - x_{ki} \ell_{ki*}) \\ = \sum_{j:i \rightarrow j} \tilde{Q}_{ij} - \sum_{k:k \rightarrow i} (\tilde{Q}_{ki} - x_{ki} \tilde{\ell}_{ki}) \end{aligned} \quad (\text{B18})$$

2) If $Q_{cci*} + \tilde{Q}_{ci} - Q_{ci*} \leq 0$, then $Q_{Li*} - Q_{cci*} - \tilde{Q}_{ci} + Q_{ci*} \geq Q_{Li*}$. According to (B5) and (B6), we have $\tilde{Q}_{cci} = 0$ and $\tilde{Q}_{Li} = Q_{Li*} - Q_{cci*} - \tilde{Q}_{ci} + Q_{ci*}$. Starting from the left side of (36) for $\tilde{\mathbf{y}}$, we have

$$\begin{aligned} \tilde{Q}_{Gi} - \tilde{Q}_{Li} - \tilde{Q}_{ci} + \tilde{Q}_{cci} \\ = Q_{Gi*} - \tilde{Q}_{ci} - (Q_{Li*} - Q_{cci*} - \tilde{Q}_{ci} + Q_{ci*}) \end{aligned}$$

$$\begin{aligned}
&= \sum_{j:i \rightarrow j} Q_{ij*} - \sum_{k:k \rightarrow i} (Q_{ki*} - x_{ki} \ell_{ki*}) \\
&= \sum_{j:i \rightarrow j} \tilde{Q}_{ij} - \sum_{k:k \rightarrow i} (\tilde{Q}_{ki} - x_{ki} \tilde{\ell}_{ki}) \quad (\text{B19})
\end{aligned}$$

Hence, $\tilde{\mathbf{y}}$ satisfies (29) and (36) in both cases.

As shown above, $\tilde{\mathbf{y}}$ satisfies (10)–(12), (29), (36), and (45), and it fulfils (44). Hence, $\tilde{\mathbf{y}}$ is a feasible point for the ADOL-socr. Because f is independent of ℓ_{di} , v'_{si} , v''_{si} , Q_{ci} and Q_{cci} for $\forall i \in \mathbb{N}^{\text{conv}}$, $\tilde{\mathbf{y}}$ yields the same objective value as $\mathbf{y}_*$. Therefore, $\tilde{\mathbf{y}}$ is also an optimal solution of the ADOL-socr. Since $\tilde{\mathbf{y}}$ attains equality in (44) and (45), it is also an optimal solution of the ADOL-ar.

This completes the proof.

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